A graph theoretical approach for exploring a board game's complexity

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Abstract: In cognitive psychology, human problem solving has been an active field of research over the past decades. During this period, methods in computer sciences and network analysis have been greatly developed. Looking at psychology's challenging questions from a computer scientist's point of view proves to be worthwhile for both disciplines. In this work, we analyze a single-player puzzle called *Rush Hour* which is solved by moving entities on a board. In a graph theoretical approach, each distinct combination of entities and their location can be represented as a node and an entity's position change can be understood as an edge. This enables us to develop complexity metrics for the popular board game *Rush Hour*, which we review by using algorithmic approaches to understand the producer's difficulty classifications. The present paper, however, concentrates on the results of a conducted study involving 97 subjects. Based on the findings, we investigate complexity measures, the subjects's navigation through the problem space as well as the game's perceived complexity. On the one hand, the results of the analysis suggest a more finely nuanced grading by difficulty for the *Rush Hour* levels. On the other hand, they contribute to a better understanding of the human perception of complexity.

1 Introduction

It is well known that problem solving capabilities of humans and computers differ in several aspects: While human can make use of their experiences, creativity, and some kind of intuition, computers must rely on the given data and algorithms in which not all real world constraints may be implemented. On the other hand, in processing and storing a big amount of information and dependencies, computers clearly outperform humans. Hence, it might be a promising approach to combine the structural advantages of human and artificial problem solving abilities in order to construct human-computer cooperative and interactive systems [AAL$^+$00]. In order to divide subtasks between human and computer agents, it is necessary to better understand why some subtasks may be a challenge to solve for both. In computer sciences, complexity theory has been providing a broad range of results about problems' difficulty for being solved by algorithms. However, in cognitive sciences, there are only a few approaches to systematically analyze a problem's complexity for humans to solve it ([RSF12], [KHS85], [HWP98]).

The present work is aiming to investigate a simple puzzle's complexity by graph theoretical methods. By considering the game's problem space as a graph, network analytic metrics are defined and the correlation between the metrics and the manufacturer's difficulty rating is analyzed, hoping to be able to classify game instances by problem space based measures into distinct complexity groups.

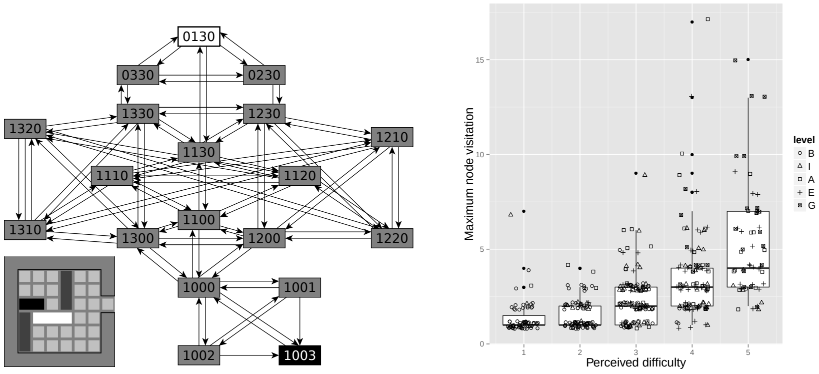
However, the present article primarily describes the findings of a conducted experiment in which the participants played some of the studied games which were selected due to their complexity metrics. The results' analysis reveal essential flaws in human problem solving abilities which could be compensated by a computer-aided system.

2 Backgrounds

Anderson defines problem solving as "goal-oriented sequence of cognitive operations" that transforms a present state into a desired state [And80]. From this definition, the concept of a problem space arises almost immediately. A problem space is defined as a set of problem states and set of operators such that an operator transforms two states into each other. Sequential application of operators yields a path through the problem space. Then, a problem belonging to a problem space can be considered as a set of start states, a set of goal states and a set of path constraints. Therefore, problem solving consists of the task to find a path through the problem space from a start to a goal state without violating the path constraints which can be seen as a searching task.

We are going to consider the sliding block puzzle game *Rush Hour* which takes place on a grid of 6×6 cells, representing a parking lot, with one exit (cf. figure 1(a)). Cars of width 1 and length 2 respective 3 cells are placed on the board vertically or horizontally and can be moved forwards or backwards as long as the for the movement needed cells are not occupied by any other car. Cars cannot move sideways and are not allowed to change their row or column, respectively. Given a configuration of cars placed on the grid, the goal is to find a sequence of moves that allows a particular car (in figure 1(a) the black one) to be moved from the board through the designated exit.

We consider the problem space of a Rush Hour game as the graph $G = (V, E)$ with V the set of all game configurations reachable by allowed moves, $E \subseteq V \times V$ the set of possible moves. The manufacturer of the Rush Hour game provides five game card sets with start configurations of five different levels of difficulty. Leaving identical configurations and configurations with a slightly different goal aside, 173 start configurations with the manufacturer's rating of complexity are available. Implementing the simple game logic and a breadth first search from every start configuration, the problem spaces were explored and, based on theoretical considerations, 24 configurations were selected for experimental analysis.



(a) **(b)**

Figure 1: (a) shows the problem space for the configuration shown. The number in the nodes correspond to the positions of the cars in their row or column: the first/second/third/fourth digit corresponds to the black/white/left grey/right grey car's position. White node is the start node, black node a solution node.(b) is the maximum node visitation of all players and all games against its perceived difficulty. Shape represents the producer's difficulty estimation (beginner, intermediate, advanced, expert, grand master)

3 Experiment

Theoretical considerations which were not described in this article, suggested that the levels' complexity statement is only based on the number of needed moves. In order to investigate if the true complexity depends on further factors, an experiment was conducted in which 74 participants played at least six of 24 selected Rush Hour games such that every configuration was played by at least 20 participants. The participants played the game online, but it was made sure that no participant played one game more than once. When a player successfully finished a game, she or he was asked to grade it by difficulty. For the analysis, only data sets of completed games were used.

The results show, as expected, the strong influence of needed moves to solve the game on the its perceived difficulty, but indicate that there exist other factors contributing to complexity. Therefore, we were interested in how the participants navigated in the game's problem space while solving the game. It is remarkable that the majority takes exactly the same way through the problem space which is not necessarily the shortest one. This finding is consistent with the observation from cognitive sciences that humans use the same heuristics for solving problems. Additionally, we could find a dependency of a player's difficulty estimation and her individual navigation through the problem space: the more a player "becomes lost" in the problem space, the more difficult the game is perceived. Modeling a player losing her orientation can be done by several approaches: If a player gets lost, her way through the problem space will certainly be longer than required, and indeed, a strong correlation between the perceived difficulty of a game and the quotient of the number of used moves to the number of moves in the optimal solution can be found. Surprisingly, in games rated by the players as very hard, the players need in average five times as many moves than it would be necessary in the optimal solution. Even in the games rated as very easy, the players need in average about 1.2 times as many moves as the optimal solution takes. Another approach considers configurations which occur several times in a player's solution (clearly not optimal). For every node in the solution of a player, define the *node visitation* as the number of how often this player uses that particular node in her solution. Maximizing over all nodes of a player's solution yields the *maximum node visitation* whose correlation to the perceived difficulty is depicted in figure 1(b). This leads to the conclusion that a problem's complexity does not only depend on objective properties, but there is also a high correlation with the individual performance. The values of the maximum node visitation take on surprisingly high values indicating that getting lost in a huge problem space is a general issue in human problem solving. But this could, though, be easily avoided by computer support, since recognizing a repeating configuration can be done algorithmically without need of completely solving or even knowing the problem. Thus, this analysis of a problem which may seem artificially constructed leads to the suggestion of the following human-computer cooperative system: the human can make use of intuition, creativity, and heuristics to solve the problem with a problem space which may be too large for a purely algorithmical solution, and the computer gives notice of repeating configurations, based on local computations, pointing the human to the right direction.

4 Outlook

There are several approaches that could be promising for further work. The observations that the resulting problem spaces are surprisingly large and that not all contained states may be relevant, as it is not necessarily important in which order moves are taken, leads to the thought whether one can merge equivalent states and therefore reduce the problem space. Reduced problem spaces could be worthwhile to consider. Furthermore, it would be an interesting question how repeating configurations in a human solution can be recognized and stored efficiently without computing the complete problem space or consider every used configuration.

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